36. $x = 3\cos t \quad y = 4\sin t \quad [0, 2\pi]$

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \text{arc length for parametric eqns}$$

$$\int_0^{2\pi} \sqrt{(-3\sin t)^2 + (4\cos t)^2}\, dt$$

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15. $x = \ln(2t) \quad y = \ln(3t)^4 = 4\ln(3t)$

$$\frac{dx}{dt} = \frac{1}{2t}\cdot 2 = \frac{1}{t} \qquad \frac{dy}{dt} = 4\cdot\frac{1}{3t}\cdot 3 = \frac{4}{t}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{4}{t}}{\frac{1}{t}} = \frac{4}{t}\cdot\frac{t}{1} = 4$$

$$\frac{d^2y}{dx^2} = 0$$

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9. $x = -\sqrt{t+1}$ $y = \sqrt{3t}$

$\frac{dx}{dt} = -\frac{1}{2}(t+1)^{-\frac{1}{2}} = \frac{-1}{2\sqrt{t+1}}$ $\frac{dy}{dt} = \frac{1}{2}(3t)^{-\frac{1}{2}} \cdot 3 = \frac{3}{2\sqrt{3t}}$

$\frac{dy}{dx} = \frac{3}{2\sqrt{3t}} \cdot \frac{2\sqrt{t+1}}{1} = -3\sqrt{\frac{t+1}{3t}}$

$\frac{d^2y}{dx^2} = \frac{-3\frac{1}{2}\left(\frac{t+1}{3t}\right)^{\frac{1}{2}} \cdot \frac{3t\cdot 1 - (t+1)\cdot 3}{(3t)^2}}{\frac{-1}{2\sqrt{t+1}}}$

$= -\frac{3}{2}\sqrt{\frac{3t}{t+1}} \cdot \frac{-3}{(3t)^2}(-2)\sqrt{t+1}$

$= \sqrt{\frac{3t}{t+1}} \cdot \frac{-\sqrt{t+1}}{t^2} = -\frac{\sqrt{3t}}{t^2}$

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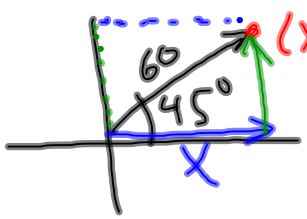
10.2 Vectors

length = magnitude

direction

polar: 60 mph, NE (45° E of N)



$60\cos 45 = \frac{x}{60}$

components $(x, y) = (60\cos 45°, 60\sin 45°)$

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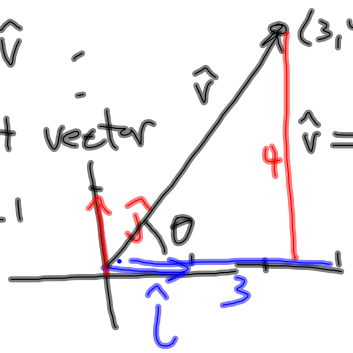
$\vec{v}_1 = (x_1, y_1)$ $\vec{v}_2 = (x_2, y_2)$

$\vec{v}_1 + \vec{v}_2 = \vec{v}_3 = (x_1 + x_2, y_1 + y_2)$

or $\hat{v}$: unit vector

length = 1



$\hat{v} = 3\hat{i} + 4\hat{j} = (3,4)$

$= \langle 3,4 \rangle$

length $= |\hat{v}| = \sqrt{3^2 + 4^2}$

$|\hat{v}| = 5$

$\theta = \tan^{-1}\frac{4}{3}$

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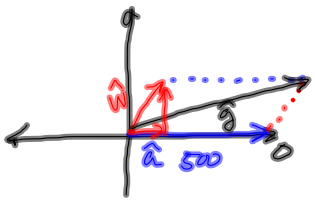
Ex 4 $\hat{a} =$ 500 mph $\angle$ 0

↑ air speed (speed in still air)



$\hat{w} =$ 70 mph $\angle$ 60°

$\hat{g} = \hat{a} + \hat{w} = \langle 500 + 70\cos 60°, 0 + 70\sin 60° \rangle$

ground speed $= \langle 535, 35\sqrt{3} \rangle$

how fast? $= |\hat{g}| = \sqrt{535^2 + 35^2 \cdot 3}$

$= 538.4$ mph

$\theta = \tan^{-1}\left(\frac{35\sqrt{3}}{535}\right) = 6.5°$

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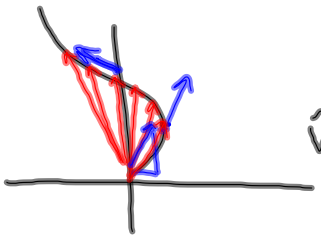
vector valued function

Position Vector $= \hat{r}(t) = \sin(t)\,\hat{i} + \frac{t^2}{2}\,\hat{j}$

$= \langle \sin(t), \frac{t^2}{2} \rangle$

path: $x = \sin t$

$y = \frac{t^2}{2}$



$\hat{r}(0) = \langle 0, 0 \rangle = 0\hat{i} + 0\hat{j}$

$\hat{r}\left(\frac{\pi}{6}\right) = \left\langle \frac{1}{2}, \frac{\pi^2}{6^2 \cdot 2} \right\rangle$

$\hat{v} = \hat{r}'(t) = \langle \cos t, t \rangle$

$= \cos t \cdot \hat{i} + t \cdot \hat{j}$

$\hat{v}\left(\frac{\pi}{3}\right) = .5\hat{i} + 1.05\hat{j}$

$\hat{a} = \langle -\sin t, 1 \rangle$

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distance traveled = arc length

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

x component of velocity

y component of velocity

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = |\hat{v}|$$

speed

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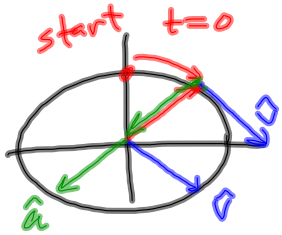
Ex 7. $\hat{r}(t) = \langle 4\sin t, 2\cos t \rangle$



$\hat{v}(t) = \langle 4\cos t, -2\sin t \rangle$

$\hat{a}(t) = \langle -4\sin t, -2\cos t \rangle$

$\hat{a} = -\hat{r}$

$\hat{r}\left(\frac{\pi}{4}\right) = \left\langle 4\frac{\sqrt{2}}{2}, 2\frac{\sqrt{2}}{2} \right\rangle = \langle 2.8, 1.4 \rangle$

$\hat{v}\left(\frac{\pi}{4}\right) = \left\langle 4\frac{\sqrt{2}}{2}, -2\frac{\sqrt{2}}{2} \right\rangle = \langle 2.8, -1.4 \rangle$

$\hat{a}\left(\frac{\pi}{4}\right) = \langle -2.8, -1.4 \rangle$

Read ex. 8

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